Published by Institute of Physics Publishing for SISSA/ISAS

Received: *November 15, 2004*
Accepted: *November 24, 2004*

Locally localized gravity and geometric transitions

Dionisio Bazeia,[a] **Francisco A. Brito**[b] **and Adalto Rodrigues Gomes**[ac]

[a]*Departamento de Física, Universidade Federal da Paraíba*
Caixa Postal 5008, 58051-970 João Pessoa, Paraíba, Brazil
[b]*Departamento de Física, Universidade Federal de Campina Grande*
58109-970 Campina Grande, Paraíba, Brazil
[c]*Departamento de Ciências Exatas*
Centro Federal de Educação Tecnológica do Maranhão
65025-001 São Luís, Maranhão, Brazil
E-mails: bazeia@fisica.ufpb.br, fabrito@df.ufcg.edu.br,
argomes@fisica.ufpb.br

Abstract: In this paper we analyze the local localization of gravity in AdS_4 thick brane embedded in AdS_5 space. The 3-brane is modelled by domain wall solution of a theory with a bulk scalar field coupled to five-dimensional gravity. In addition to small four-dimensional cosmological constant, the vacuum expectation value (vev) of the scalar field controls the emergence of a localized four-dimensional quasi-zero mode. We introduce high temperature effects, and we show that gravity localization on a thick 3-brane is favored below a critical temperature T_c. These investigations suggest the appearance of another critical temperature T_*, where the thick 3-brane engenders the geometric $AdS/M/dS$ transitions.

Keywords: Field Theories in Higher Dimensions, Classical Theories of Gravity.

JHEP11(2004)070

Contents

1. Introduction

The localization of gravity on a brane [1] has appeared as an alternative to compactification involving infinite extra dimension. This realization can be considered in a five-dimensional gravity theory, with a negative cosmological constant Λ_5, where we can obtain a 5d anti-de-Sitter (AdS_5) solution. When a 3-brane is introduced, gravity can be localized on it. In the Randall-Sundrum [1, 2] scenarios, 3-branes are embedded in AdS_5 bulk space where one considers a five-dimensional gravity with negative cosmological constant Λ_5 and source of "infinitely thin" 3-branes given by delta functions. As it was shown, there is a *perfect* fine-tuning between branes tension and the cosmological constant Λ_5. This fine-tuning leads to a 4d Minkowski (M_4) brane with four-dimensional cosmological constant $\Lambda = 0$, such that only the AdS_5 space is curved. The graviton zero mode bound to the 3-brane is responsible for a 4d localized gravity. The correction to the newtonian potential due to Kaluza-Klein gravitons is highly suppressed, at low energy. On the other hand, if perfect fine-tunings are absent then both 3-brane and AdS_5 space can be curved. These branes are either 4d de Sitter (dS_4) branes with $\Lambda > 0$ or 4d anti-de-Sitter (AdS_4) branes with $\Lambda < 0$. Explicit solutions of AdS_4, dS_4 and M_4 branes were put forward in [3]–[9].

The issue of local localization of gravity on AdS_4 branes was firstly addressed in [9] — see also [10, 11, 12] for connection between massive gravity in dS_4 and AdS_4 space and absence of van Dam-Veltman-Zakharov discontinuity. The graviton mode responsible for the 4d gravity is not a zero mode but an almost massless mode, the "quasi-zero" mode that dominates over the Kaluza-Klein modes. This is a far more general mechanism of localization of gravity, because it does not require any condition on the space far from the brane. Under this perspective, gravity localization can be realized in string theory, once no-go theorems about localization in supergravity theories, e.g. [13, 14, 15] relying on the asymptotic behavior of the geometry do not necessarily apply [9]— see [16] for further discussions. There are partial supersymmetric realizations of massless [17] and massive [18] gravity localization in braneworld scenarios, which arise from a sphere reduction in string/M-theory. Here, only the bulk lagrangian can be viewed as arising from a higher dimensional supergravity while the 3-brane is supported by a delta function put by hand. However, as one considers non-homogeneous quaternionic moduli spaces, a complete 5d supergravity realization of gravity localization can be incorporated smoothly by thick 3-branes supported by scalar fields [19]. Thus, under this perspective it is useful to study models where thick brane solutions exist, and gravity can also be trapped.

Universal aspects of localization of gravity in *thick* M_4 branes were first studied in [20]. Scalar fields to model such thick branes were introduced in [8, 20, 21]. Thick dS_4 and AdS_4 branes which use bulk scalar fields have also been considered in the literature. Models implementing a scalar field with scalar potentials like $\cos^n(\phi)$ that can be solved analytically have been investigated in [22]–[27] in different contexts. A $\lambda\phi^4$ model was explored analytically in [8] in first order formalism, but just in the $\Lambda = 0$ case. In ref. [28] this model was also considered in the context of local localization of gravity, but the solutions were found only in the thin wall limit.

In this paper we consider a $\lambda\phi^4$ model to investigate the local localization of gravity on *thick* AdS_4 branes. We use numerical methods to solve the equations of motion and to obtain the graviton spectrum of gravity fluctuations around the brane solution. We explore in detail how the lightest graviton mode binds to the brane. The scalar field vev and the coupling constant λ control the emergence of this graviton. We also consider high temperature effects in the bulk and discuss how they can affect the localization of gravity. Our motivation is to address the issue of geometric transitions, where the 3-brane changes from AdS_4 to M_4, and then to dS_4 as the temperature diminishes. This mechanism occurs on a thick 3-brane, lifting a supersymmetric vacuum $\Lambda < 0$ to another one, non-supersymmetric, with $\Lambda > 0$. This is a current discussion in string theory; see, e.g., the investigation introduced in ref. [29]. A small and positive 4d cosmological constant agrees with current observational data, which show that our universe experiences an accelerated expansion [30, 31].

The paper is organized as follows. In section 2 we introduce the model and the formalism applied to our analysis. In section 3 we discuss how brane solutions appear in a theory engendering 5d gravity and scalar field in the thin wall limit. In section 4 we extend the analysis of the previous section to the study of thick branes and high temperature effects. We end the paper in section 5 where we present our final considerations.

2. Preliminaries

We consider the model with five-dimensional gravity coupled to a scalar field

$$S = \int d^5x\sqrt{g}\left[-\frac{1}{4}R + \frac{1}{2}\partial_M\phi\partial^M\phi - V(\phi)\right], \tag{2.1}$$

where we consider the signature $(+ - - - -)$ and $M = 0, 1, 2, 3, 4$, with $g = \det(g_{MN})$.

The metric Ansatz is

$$ds^2 = e^{2A(r)}\bar{g}_{\mu\nu}dx^\mu dx^\nu - dr^2\,, \tag{2.2}$$

where $\bar{g}_{\mu\nu}$ is the four-dimensional metric, with $\mu,\nu = 0,1,2,3$, satisfying

$$\bar{R}_{\mu\nu} = -3\Lambda\bar{g}_{\mu\nu}\,. \tag{2.3}$$

The four-dimensional cosmological constant Λ is positive for de Sitter (dS_4) spacetime, negative for anti-de Sitter (AdS_4) spacetime and zero for Minkowski (M_4) spacetime.

We are mainly interested in localization of gravity on a 3-brane. This requires the study of gravity fluctuations around the brane solution [1, 8, 9, 20]. On this account we linearize the Einstein equations by considering a perturbation as $\bar{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$. Taking into account only the traceless transverse (TT) sector of the linear perturbation, where gravity equations of motion do not couple to matter fields, we find $\partial_M(\sqrt{g}g^{MN}\partial_N)\Phi = 0$ or in terms of metric components

$$\left(\partial_r^2 + 4A'\partial_r - e^{-2A}(\Box_{4d} + 2\Lambda)\right)\Phi = 0\,, \tag{2.4}$$

where Φ describes the wave function of the graviton on non-compact coordinates. Let us consider the Ansatz $\Phi = h(r)M(x^\mu)$, the equation describing the 4d graviton $(\Box_{4d} + 2\Lambda)M = m^2 M$ and the change of variable $h(r) = e^{3A(z)/2}\psi(z)$, $z(r) = \int e^{-A(r)}dr$ into eq. (2.4). In this way we get to the Schroedinger-like equation

$$-\,\partial_z^2\psi(z) + U(z)\psi(z) = m^2\psi(z)\,, \tag{2.5}$$

with the potential

$$U(z) = \frac{9}{4}A'(z)^2 + \frac{3}{2}A''(z)\,. \tag{2.6}$$

3. Brane solutions in the thin wall limit

To show how a Z_2-symmetric 3-brane arises through the scalar field in the thin wall limit [8, 32], we consider the equations of motion

$$\phi'' + 4A'\phi' = \frac{\partial V(\phi)}{\partial\phi} \tag{3.1}$$

$$A'' + \Lambda e^{-2A} = -\frac{2}{3}\phi'^2 \tag{3.2}$$

$$A'^2 - \Lambda e^{-2A} = -\frac{1}{3}V(\phi) + \frac{1}{6}\phi'^2 \tag{3.3}$$

where we assume that the scalar field only depends on the extra dimension, r. For non-zero cosmological constant Λ, the integrability of these equations through first order formalism [8] is hard. For this reason, we work with the second order equations.

JHEP11(2004)070

In our analysis, the scalar field is a typical kink approaching the asymptotic values $\phi(r \to \pm\infty) \to \pm a$. We choose the kink solution and the Z_2-symmetric scalar potential as

$$\phi = a \tanh \lambda a r \tag{3.4}$$

$$V(\phi) = \frac{1}{2}\lambda^2(\phi^2 - a^2)^2 - \frac{3}{L^2}\,, \tag{3.5}$$

where $V(\pm a) \equiv \Lambda_5 = -3/L^2$ is identified with the AdS_5 cosmological constant. We can quickly check that this cannot even satisfy the equation (3.1) because of the term $4A'\phi'$. However, this is not true in the *thin wall limit*. In this limit ($\lambda \to \infty$ and $a \to 0$, with λa^3 fixed) we can approach the kink solution to a step function $\phi \simeq a\,\mathrm{sgn}(r)$ whose width $\Delta \simeq 1/\lambda a$ goes to zero. This provides the "identities"

$$\phi' \simeq 2a\delta(r)\,, \qquad \phi'^2 \simeq \sigma\delta(r)\,. \tag{3.6}$$

In our five-dimensional set up, σ is identified with a positive 3-brane tension given by

$$\sigma = \frac{4}{3}\lambda a^3\,. \tag{3.7}$$

This is precisely the kink energy in absence of gravity. As we shall see later, we can also use this formula for thick branes coupled to 5d gravity. This is because the back reaction on the kink by turning on gravity is very small.

We now turn our attention to equations (3.1)–(3.3). In the thin wall limit $A'\phi' \to 2aA'(r)\delta(r)$, which is zero everywhere provided the function $A(r)$ satisfies the boundary condition $A'(0) = 0$. In this limit, the equation (3.1) is satisfied and the two other equations now reads

$$A'' + \Lambda e^{-2A} = -\frac{2}{3}\sigma\delta(r) \tag{3.8}$$

$$A'^2 - \Lambda e^{-2A} = \frac{1}{L^2}\,. \tag{3.9}$$

Note that the scalar field contributions in (3.3) cancel out, because of their consistency with (3.1). The equations above could be obtained from the action

$$S = \int d^5x\sqrt{g}\left[-\frac{1}{4}R + \frac{3}{L^2}\right] - \sigma\int d^4x dr\sqrt{g}\,\delta(r)\,, \tag{3.10}$$

which is the thin wall limit of the action (2.1). It describes two copies of AdS_5 with curvature $\Lambda_5 = -3/L^2$ pasted together along an "infinitely thin" 3-brane located at $r = 0$ with tension σ.

It is not difficult to check that the equations (3.8)–(3.9) are satisfied by the well known solutions [3]–[9]

$$dS_4\ (\Lambda > 0):\qquad A(r) = \ln\left(\sqrt{\Lambda}L\sinh\frac{c-|r|}{L}\right), \qquad \sigma = \frac{3}{L}\coth\frac{c}{L} \tag{3.11}$$

$$M_4\ (\Lambda = 0):\qquad A(r) = \frac{c-|r|}{L}\,, \qquad \sigma = \frac{3}{L} \tag{3.12}$$

$$AdS_4\ (\Lambda < 0):\qquad A(r) = \ln\left(\sqrt{-\Lambda}L\cosh\frac{c-|r|}{L}\right), \qquad \sigma = \frac{3}{L}\tanh\frac{c}{L}\,, \tag{3.13}$$

where c is a real constant. As we mentioned earlier, the perfect fine-tuning $\sigma = 3/L = L|\Lambda_5|$ in the M_4 brane imposes $\Lambda = 0$. By making $c \to \infty$ in the other fine-tuning equations, both dS_4 and AdS_4 branes collapse to M_4 branes. This is precisely the fine-tuning imposed in the Randall-Sundrum scenario [2]. The 3-brane tension is now given explicitly in terms of the vev a of the scalar field via equation (3.7). There exists an analog of this picture in four-dimensional supergravity domain walls. The M_4 brane with perfect fine-tuning, i.e., $c \to \infty$, is analogous to a 4d BPS saturated domain wall ($\sigma = \sigma_{\rm BPS}$) while the dS_4 and AdS_4 branes with c finite are analogous to 4d "bent walls" whose tension are $\sigma > \sigma_{\rm BPS}$ and $\sigma < \sigma_{\rm BPS}$, respectively — for a review see [33, 34].

The physics of such solutions as gravity localization on AdS_4 branes was first explored in [9] — see also [35, 36]. As it was stressed, this is the scenario where *locally localized gravity* emerges. This phenomenon is local, because in the solution (3.13), it is only around $|r| \leq c$ that the warp factor approaches that of a M_4 brane. For $|r| \gg c$ the space include the AdS boundary, so it has infinite volume. By showing that gravity localization is a local effect, we avoid issues concerning the global behavior of the extra dimension. Furthermore, since the volume of extra dimension is infinite in this case, the normalized graviton bound to the brane cannot be a massless one. In fact, as it has been shown, the graviton bound to the brane is an almost massless graviton [9] — for phenomenological issues on extra dimensions with infinite volume see refs. [37, 38, 39].

In the following sections, we search for thick brane solutions and explore how this physical scenario emerges in our model.

4. Thick brane solutions

In this section, we relax the limit of zero thickness considered above to find how thick branes can arise via the scalar field. We shall essentially look for a smooth version of the "infinitely thin" brane solutions (3.11), (3.12) and (3.13) and study how they can localize gravity. We shall mainly focus on gravity localization in AdS_4 branes. For thick branes, however, due to difficulties to integrate eqs. (3.1)–(3.3) analytically, we content ourselves with numerical solutions.

We deal mainly with eqs. (3.1)–(3.2), while eq. (3.3), as an energy conservation check, give us Λ_5. The boundary conditions $\phi(0) = 0$, $\phi(\infty) = a$ and $A(0) = 0$, $A'(0) = 0$ suggest that we use the finite difference method for $\phi(r)$, and Runge-Kutta method for $A(r)$. We propose the kink profile $\phi_1(r) = a\tanh(\lambda a r)$ as a first tentative solution. Then we use $\phi_1(r)$ in eq. (3.2) and apply Runge-Kutta method to obtain $A_1(r)$. Next, we transform eq. (3.1) in a set of N coupled nonlinear algebraic equations, with $A_1(r)$ given, and with N values $\phi_2(r)$ to be determined. The procedure goes on until a desired precision to obtain $\phi(r)$ and $A(r)$ is achieved [40, 41, 42]. Then, we use eq. (3.3) to get Λ_5. As an example, for $\lambda = a = 1$ and $\Lambda = -0.2$, as we go further with the iterations, eq. (3.3) leads Λ_5 to the value -0.77.

The parameters a and λ are specified according to the thickness and tension of the brane. The cosmological constant Λ is also specified with the scenario we want to study, i.e., AdS_4, dS_4 or M_4. In figure 1 and in figure 2 we show the behavior of the solutions

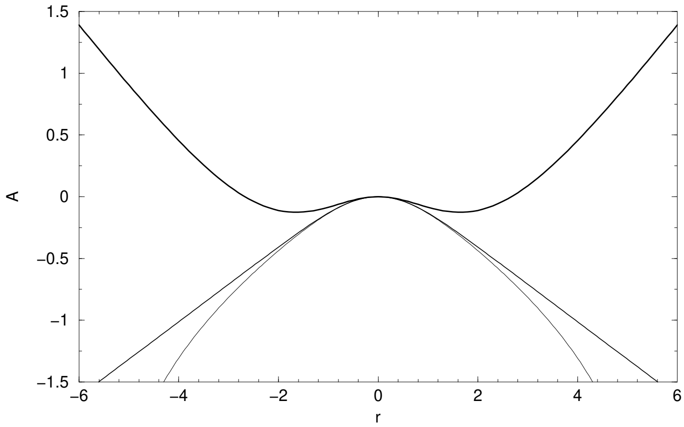

Figure 1: The solution $A(r)$ for $\Lambda = -0.2$ (thicker curve), $\Lambda = 0$, and $\Lambda = +0.02$ (thinner curve).

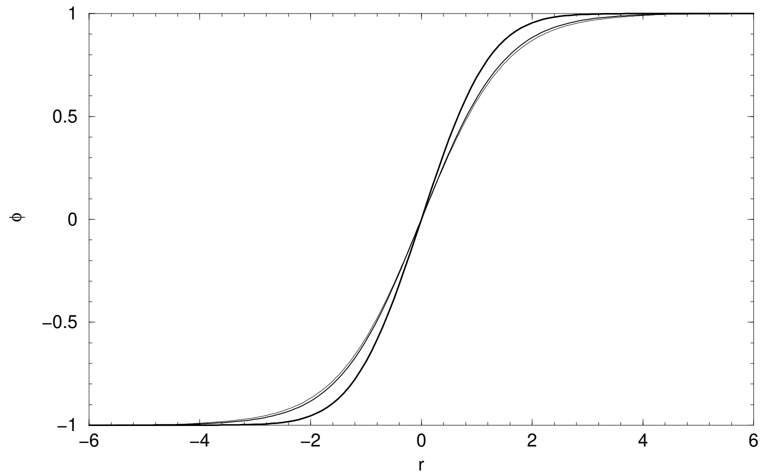

Figure 2: The kink solutions $\phi(r)$ for $\Lambda = -0.2$, 0, and $+0.02$. They almost confuse in the last two cases.

$A(r)$ and the kink profile $\phi(r)$, respectively, for $\Lambda = -0.2$, 0, $+0.02$. Note that the kink profiles are almost the same regardless of the value of Λ, with slightly smaller thickness in the AdS_4 case. The tension can be related to Λ and to Λ_5 via fine-tuning [1, 2, 9, 33, 34]. Indeed, there exists a function that can be determined numerically engendering a fine-tuning $\sigma = (3/L)f(c)$. For $\lambda = a = 1$ we find $f(c) = 0.87$, where $c \simeq 1.6$ for $\Lambda = -0.2$. A finite constant c characterizes the solution $A(r)$ for non-zero 4d cosmological constant — see the solutions in eqs. (3.11) and (3.13). As also happens in infinitely thin branes [9], for dS_4 branes c is the distance between the brane and the horizon, whereas for AdS_4 branes c is the distance to the turn around point in $A(r)$ — see figure 1. The latter case is of particular interest. The solution diverges for $r \gg c$ approaching the boundary of the AdS_5, but close to the brane ($r \lesssim c$) it behaves like a M_4 brane. Thus, it is expected that gravity is locally localized on the 3-brane. The bound graviton, however, is not a massless graviton, but an almost massless graviton. In fact, this graviton is the lightest mode of an infinite Kaluza-Klein tower of states as the Schroedinger-like potential for $\Lambda = -0.2$

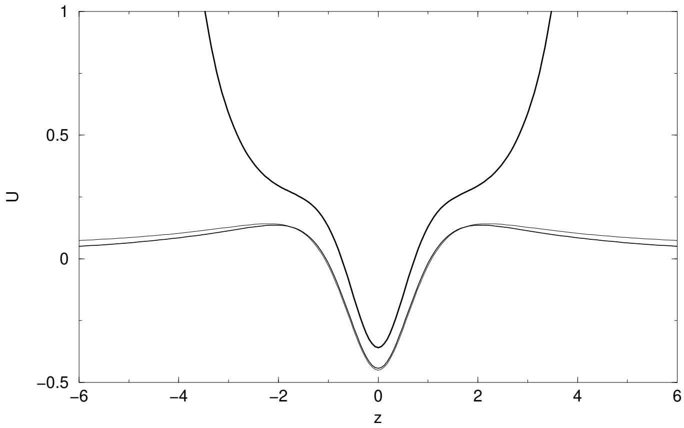

Figure 3: The Schroedinger-like potential for $\Lambda = -0.2$, 0, and $+0.02$.

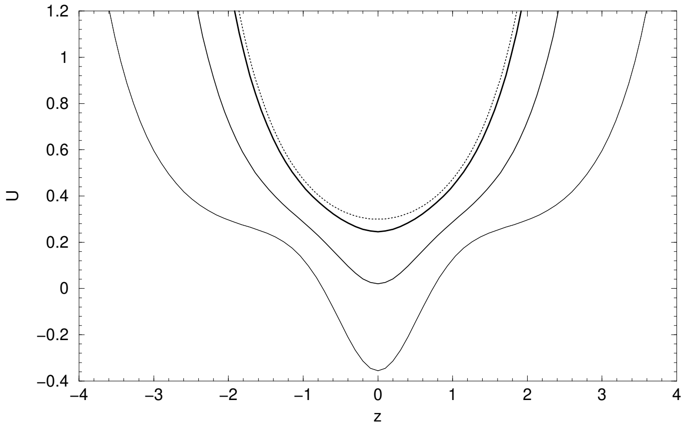

Figure 4: The Schroedinger-like potential for fixed width and tensions decreasing with $a = 1$, $5/10$ and $2/10$. For $a = 2/10$ we approach the tensionless brane limit (upper curve).

JHEP11(2004)070

indicates in figure 3. This potential essentially represents a box in contrast to the volcano-like potentials for $\Lambda = 0$ and $\Lambda = +0.02$. In the dS_4 case, the potential asymptotes to non-zero value, showing that there is a gap separating the zero mode from the continuum spectrum. In the following we turn our attention only to the AdS_4 case.

4.1 The localization of gravity in AdS_4 thick branes

The results show that as we decrease the brane tension keeping the brane thickness fixed, in the limit of tensionless brane we reproduce the spectrum of gravity fluctuations of pure AdS_5 space [9, 35, 36]. The numerical calculations below are performed using $\Lambda = -0.2$, except in some cases where we have to specify other values.

In figure 4 we show how the Schroedinger-like potential changes with the brane tension. In the tensionless brane limit we have a pure AdS_5 potential. In this limit, the solution of the quantum mechanics problem gives us the energy eigenvalues [9]

$$E = n(n+3)\,, \qquad n = 1, 3, 5\ldots \tag{4.1}$$

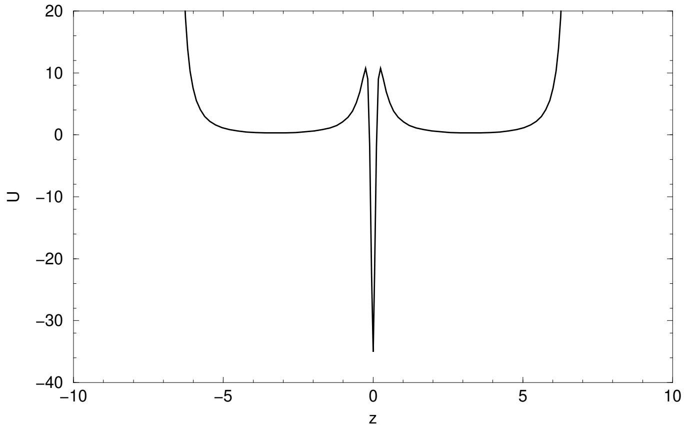

Figure 5: The Schroedinger-like potential for a brane with $\Delta = 0.1$ and tension for $a = 1.0$. The AdS_5 potential tends to split into two copies.

The squared masses are defined as $m^2 = E|\Lambda|$. The other cases are harder to investigate. However, we can find the energy spectrum numerically by searching for the zeroes of the wavefunction ψ at the boundaries of the box-like potential $U(z)$ as a function of the energy E, as suggested in ref. [9]. For each energy, the wavefunction is obtained by the Numerov method [43].

In our investigation, we have found an interesting behavior: as we increase the brane tension, one mode become almost massless in comparison with all the other modes of the spectrum. It has an amplitude on the brane much higher than all the other modes, as we shall see later explicitly. We refer to this state as the *quasi-zero* mode. This is the mode responsible for localizing gravity on the brane.

In the thin wall approximation ($\Delta = 0.1$, for instance), as we increase the tension, the Schroedinger-like potential tends to become two copies of the pure AdS_5 potential pasted together along the brane source — see figure 5. This is similar to the critical limit of brane tension found in [9, 35, 36]. The physical meaning is that now in the AdS_5 spectrum there is one quasi-zero mode trapped on the brane responding for the four-dimensional gravity.

Note that by fixing $\Delta = 1/\lambda a$, the tension $\sigma = (4/3)a^2/\Delta$ depends only on a. In this way, we can leave only the scalar field vev a as a free parameter. This parameter can be understood as an energy scale that can control the localization of gravity. As an example, for $\Delta = 1$ and several brane tensions $\sigma = (4/3)a^2$, figure 6 shows how in the tensionless brane limit the massive graviton spectrum obtained approaches the pure AdS_5 spectrum just as in the Karch-Randall scenario (crosses). For numerical reference see the table IV.A. For higher values of brane tensions, the wavefunction amplitude of the first mode is very much larger than the amplitude of the other excited states. In figure 7 we show that the lightest mode is at $a = 1$, since it occurs for greater tension; the first mode (upper black diamond) has amplitude on the brane much higher than the other modes (adjacent black diamonds). It separates from the other modes, and it is our almost massless mode that contribute to the newtonian potential as a leading term that dominates over the other Kaluza-Klein (heavier) modes. Another behavior shown in figure 7 is that as the squared

JHEP11(2004)070

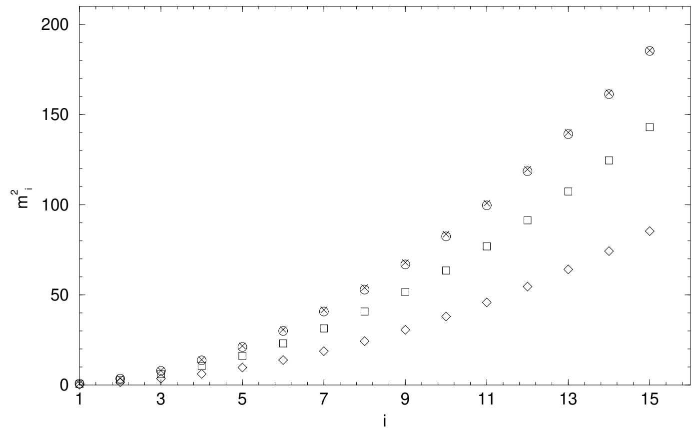

Figure 6: Graviton masses for a fixed brane thickness $\Delta = 1$ and tensions $\sigma = (4/3)a^2$ for $a = 1$ (diamonds), 5/10 (squares), 2/10 (circles) and $a = 0$ (crosses).

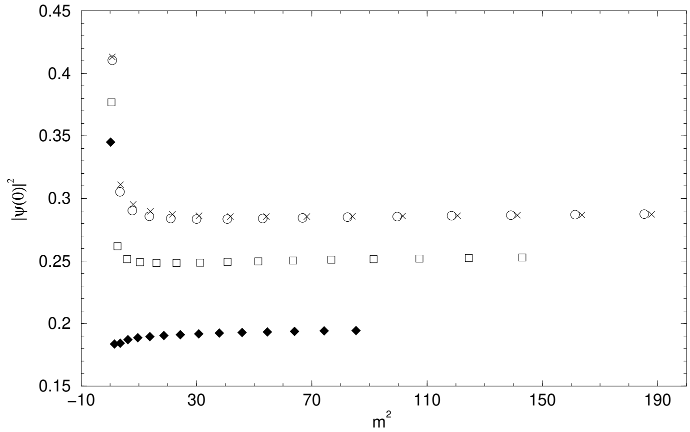

Figure 7: The wavefunction amplitude on the brane $|\psi_i(z = 0)|^2$ for the squared masses m_i^2, with brane thickness $\Delta = 1$ and tensions $\sigma = (4/3)a^2$. The plots are for $a = 1$ (black diamonds), 5/10 (squares), 2/10 (circles), and $a = 0$ (crosses).

masses m_i^2 increase, the amplitudes $|\psi_i(z = 0)|^2$ tend to constants which depend of the corresponding tensions. In fact, in this limit we have checked that the wavefunctions for the potential $U(z)$ can be very well approximated to that of a box-like potential, where $|\psi(0)|^2 = 1/z_\infty$, with z_∞ being a constant that identifies the contour of the box.

We can also see that as $|\Lambda|$ decreases, the mass squared of this graviton mode, the quasi-zero mode m_1^2 vanishes faster than all the other modes, as we show in figure 8. To illustrate this situation, we recall that the mass squared of the first four modes depends on $|\Lambda|$ according to the power-laws:

$$m_1^2 \simeq 1.83\,|\Lambda|^{1.46}\,, \qquad m_2^2 \simeq 9.80\,|\Lambda|^{1.18}\,, \qquad m_3^2 \simeq 22.20\,|\Lambda|^{1.15}\,, \qquad m_4^2 \simeq 39.0\,|\Lambda|^{1.14}. \tag{4.2}$$

See also [35, 36] for former investigations on this issue. We have checked that although the power in $|\Lambda|$ decreases for increasing i, it remains always bigger than 1. This is in agreement with the fact that in the limit $\Lambda \to 0$, we approach the M_4 case, in which

i	$a=1$	$a=5/10$	$a=2/10$	AdS_5
hline 1	0.190	0.507	0.753	0.8
2	1.493	2.618	3.474	3.6
3	3.488	5.917	7.751	8.0
4	6.190	10.417	13.592	14.0
5	9.622	16.133	21.011	21.6
6	13.784	23.082	30.025	30.8

Table 1: Graviton masses for decreasing tensions $\sigma = (4/3)a^2$ with $\Delta = 1$, $a = 1, 5/10, 2/10$. The last column is for pure AdS_5 masses $m_i^2 = n(n+3)|\Lambda|$, $n = 2i - 1$.

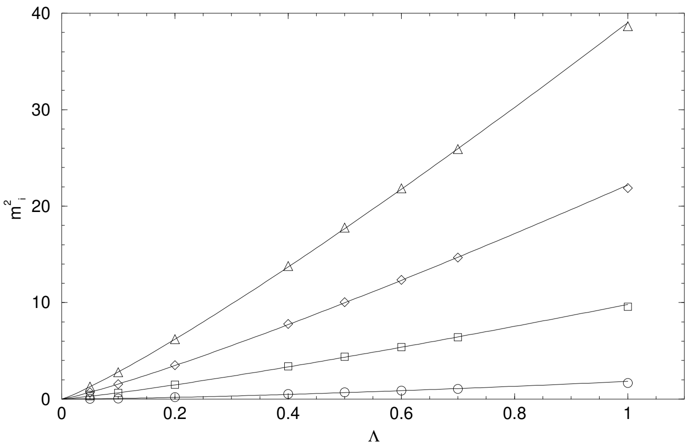

Figure 8: The square of mass of the four first graviton modes as a function of Λ for a fixed brane thickness $\Delta = 1$ and brane tension $\sigma = (4/3)$.

the brane is flat and the graviton is a zero mode [1], with no other discrete mode in the spectrum. The massless limit is smooth, and shows no van Dam-Veltman-Zakharov discontinuity [10, 11, 12]. Thus, the brane tension and 4d cosmological constant Λ are responsible for the emergence of the almost massless graviton. Similar results are obtained by fixing the brane tension and varying the brane thickness.

4.2 High temperature effects and geometric transitions

In cosmology, the cosmic evolution may be related to the temperature, which is introduced as a parameter that controls the way a phase transition can occur. In our investigation, the vev scale a that we have used can perfectly be related to high temperature effects. The 5d fields in the bulk can be regarded as 4d fields on the brane with an infinite Kaluza-Klein (KK) tower of masses. On this account one can compute the 5d effective potential at high temperature in the standard way by summing the contribution of the KK tower to the 4d effective potential [44].

As we know, the effective mass for a scalar meson at high temperature has the standard behavior [45, 46]

$$m^2(T) = m^2(0) + BT^2\,, \tag{4.3}$$

where B is a numerical coefficient that depends on the higher excited Kaluza-Klein modes [44]. In general, there are two cases to be considered: first, $m^2(0) < 0$ and $B > 0$, where the scalar potential has its symmetry restored at temperatures higher than a critical temperature T_c, and, second, $m^2(0) > 0$ and $B < 0$, where symmetry is restorated at temperatures lower than T_c. In ref. [47], it was addressed the issue of disappearance of 2d domain walls embedded in a 4d Minkowski space-time. This requires that one assumes the second case, where symmetry restoration occurs at temperatures lower than T_c. In this regime, there is no domain wall at all at low temperature, as it is required by observational data in our 4d universe. However, if we consider that our universe is itself a 3d domain wall that was formed at very high temperature, we necessarily have to follow the route that appears in the first case above.

Furthermore, in the former section we considered a domain wall with a negative cosmological constant. This 3d domain wall is what we referred to as an AdS_4 thick 3-brane. For this reason, our main goal in this subsection is to investigate how the brane tension, which should depend on the temperature, could be used to control localization of gravity. Moreover, since the cosmological constant depends on the brane tension, we shall also study how it can induce a phase transition driven by the 4d cosmological constant Λ.

The determination of the value of B is model dependent, and is out of the scope of this paper — see [44] for further details.[1] In spite of this, we can go on to see that in our model we have $m^2(0) = -2\lambda^2 a^2$; thus, at high temperature one breaks the symmetry as the temperature diminishes until $m^2(T = T_c) = 0$, where a critical temperature $T_c^2 = 2\lambda^2 a^2/B$ is found, which corresponds to the tensionless brane limit. In this limit, clearly there is no brane. By lowering the temperature to $T < T_c$, we reach the symmetry-broken phase of the scalar potential, and then a brane with tension $\sigma \sim \lambda\, a_T{}^3$ and scale $a_T = \sqrt{a^2 - BT^2/2\lambda^2}$ is recuperated. In this regime, we can find a temperature interval where the tension is large enough to favor localization of gravity.

As the temperature decreases even more, the tension becomes even larger and it is expected that in such regime another phase transition occurs, since the tension can dominate over the 5d cosmological constant. We can achieve this situation from the fine-tuning equations of section 3 — which are valid in the thin wall approximation. The information can be recast to the form [9]

$$\Lambda = \frac{1}{L^2}\left(\frac{L^2\sigma^2}{9} - 1\right). \tag{4.4}$$

This result shows that by fixing $L\Lambda_5$ for all temperatures, we have (i) $\Lambda > 0$ for $\sigma > \sigma_*$, (ii) $\Lambda = 0$ for $\sigma = \sigma_*$ and (iii) $\Lambda < 0$ for $\sigma < \sigma_*$, where $\sigma_* = L|\Lambda_5|$ is the critical tension. This give us another critical temperature

$$T_*{}^2 \simeq T_c{}^2 - \frac{2}{B}\left(\lambda^2 L|\Lambda_5|\right)^{2/3}, \tag{4.5}$$

which is clearly below the critical temperature of brane formation and gravity localization T_c. The AdS_4 brane exists and localizes gravity only in the interval $T_* < T < T_c$. For

[1] Indeed, in the region of criticality, that is, for T around T_*, we have that Λ is around zero and the results obtained in [44] for flat 3-brane can also be applied here.

$T = T_*$ only the M_4 brane is allowed, and for $T < T_*$ it is the dS_4 brane which is favored. The reasoning is valid under the high temperature approximation, so we have to care about extending it into a much lower temperature region.

We notice that the geometric transitions $AdS/M/dS$ at $T = T_*$ imply a breaking of supersymmetry on the 3-brane, since supersymmetry is not compatible with the dS_4 structure of the spacetime. This scenario seems to conform very naturally with phenomenology, and will be further investigated elsewhere.

5. Discussions

In this work, we have investigated graviton localization on a single AdS_4 thick brane, where a quasi-zero mode is responsible for 4d gravity. The brane tension increases monotonically with the vev of the bulk scalar field. For values of vev large enough an almost massless mode emerges. In this way, the vev is a scale of energy which can control localization of gravity. When we turn on the high temperature effects in the bulk, this scale becomes temperature dependent and so does the tension. Thus, there is a critical temperature T_c for which there is no brane — the tensionless brane limit. Below the critical temperature a 3-brane with tension is formed, and this favors gravity localization. This is the regime where warped compactification is established. Thus, if our universe originates at very high temperatures, and if it appears to be five-dimensional, it may become spontaneously compactified down to four dimensions below a critical temperature. In this regime, supersymmetry can play the fundamental role of imposing that only AdS_4 branes can be formed, instead of M_4 or dS_4 brane.

After reaching the critical temperature T_c for brane formation, as the temperature diminishes, the tension of the brane increases until a new phase transition occurs. This is because of the fine-tuning between the brane tension σ and the 5d cosmological constant $\Lambda_5 = -3/L^2$ that imposes conditions on the 4d cosmological constant Λ. For $\sigma \to L|\Lambda_5|$ our universe tends to a M_4 brane with cosmological constant $\Lambda \to 0$. In this regime, the tower of masses m_i^2 decreases faster than the 4d cosmological constant, according to distinct power-laws. As the 4d cosmological constant vanishes, the quasi-zero mode becomes a zero mode trapped on the M_4 brane [1]. This leads to a smooth massless limit, which implies that no van Dam-Veltman-Zakharov discontinuity exists [10, 11, 12]. The zero mode here is just at threshold of the continuum spectrum of Kaluza-Klein modes. As the temperature decreases even more, the brane tension increases until $\sigma > L|\Lambda_5|$, and our universe becomes a dS_4 brane with cosmological constant $\Lambda > 0$, with the four-dimensional graviton being a zero mode separated from the continuum spectrum. As we mentioned earlier, a small positive 4d cosmological constant agrees with current observational data which show that our universe experiences an accelerated expansion [30, 31]. Thus, the phase transition where our universe changes its geometry from an AdS_4 brane ($\Lambda < 0$) to a dS_4 brane ($\Lambda > 0$) occurs at the critical temperature T_*. The maximum value of σ occurs for $BT^2 \ll 2a^2\lambda^2$, where Λ stabilizes at some small positive constant.

The geometric phase transition which occurs at T_* connects a scenario where supersymmetry is present (AdS_4 brane) to another one, without supersymmetry (dS_4 brane).

Thus, it also indicates the presence of a soft supersymmetry breaking mechanism, which "uplifts" the 4d supersymmetric vacuum $\Lambda < 0$ to a non-supersymmetric vacuum $\Lambda > 0$.

The investigations that we have done are valid for a thick 3-brane. It can be extended to two or more 3-branes where issues such as modulus stabilization should be considered [48] — see also [44] for high temperature effects and modulus stabilization. Another issue is related to the recent investigation, where two AdS_4 3-branes with two gravitons which can vary their masses with the position of the branes [49]. Other investigations are also of interest, e.g., branes with internal structure [50, 51], critical phenomena in braneworlds [52] and gravity localization in scenarios with tachyon potentials and supergravity braneworlds [53] can be reconsidered for AdS_4 and dS_4 branes.

Acknowledgments

We would like to thank PROCAD/CAPES and PRONEX/CNPq/FAPESQ for financial support. DB thanks CNPq for partial support, FAB thanks Departamento de Física, UFPB, for hospitality, and ARG thanks FAPEMA for a fellowship.

References

[1] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[3] M. Cvetič, S. Griffies and H.H. Soleng, *Local and global gravitational aspects of domain wall space-times*, *Phys. Rev.* **D 48** (1993) 2613 [gr-qc/9306005].

[4] M. Cvetič and J. Wang, *Vacuum domain walls in d-dimensions: local and global space-time structure*, *Phys. Rev.* **D 61** (2000) 124020 [hep-th/9912187].

[5] N. Kaloper, *Bent domain walls as braneworlds*, *Phys. Rev.* **D 60** (1999) 123506 [hep-th/9905210].

[6] H.B. Kim and H.D. Kim, *Inflation and gauge hierarchy in Randall-Sundrum compactification*, *Phys. Rev.* **D 61** (2000) 064003 [hep-th/9909053].

[7] T. Nihei, *Inflation in the five-dimensional universe with an orbifold extra dimension*, *Phys. Lett.* **B 465** (1999) 81 [hep-ph/9905487].

[8] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, *Phys. Rev.* **D 62** (2000) 046008 [hep-th/9909134].

[9] A. Karch and L. Randall, *Locally localized gravity*, *J. High Energy Phys.* **05** (2001) 008 [hep-th/0011156].

[10] M. Porrati, *No van dam-veltman-zakharov discontinuity in AdS space*, *Phys. Lett.* **B 498** (2001) 92 [hep-th/0011152].

[11] I.I. Kogan, S. Mouslopoulos and A. Papazoglou, *The $m \to 0$ limit for massive graviton in dS_4 and AdS_4: how to circumvent the van Dam-Veltman-Zakharov discontinuity*, *Phys. Lett.* **B 503** (2001) 173 [hep-th/0011138].

[12] I.I. Kogan, S. Mouslopoulos and A. Papazoglou, *A new bigravity model with exclusively positive branes*, *Phys. Lett.* **B 501** (2001) 140 [hep-th/0011141].

[13] R. Kallosh and A.D. Linde, *Supersymmetry and the brane world*, *J. High Energy Phys.* **02** (2000) 005 [hep-th/0001071].

[14] K. Behrndt and M. Cvetič, *Anti-de Sitter vacua of gauged supergravities with 8 supercharges*, *Phys. Rev.* **D 61** (2000) 101901 [hep-th/0001159].

[15] J.M. Maldacena and C. Núñez, *Supergravity description of field theories on curved manifolds and a no-go theorem*, *Int. J. Mod. Phys.* **A 16** (2001) 822 [hep-th/0007018].

[16] A. Karch and L. Randall, *Localized gravity in string theory*, *Phys. Rev. Lett.* **87** (2001) 061601 [hep-th/0105108].

[17] M. Cvetič, H. Lu and C.N. Pope, *Domain walls with localised gravity and domain-wall/QFT correspondence*, *Phys. Rev.* **D 63** (2001) 086004 [hep-th/0007209].

[18] J.F. Vazquez-Poritz, *Massive gravity on a non-extremal brane*, *J. High Energy Phys.* **12** (2001) 030 [hep-th/0110299].

[19] K. Behrndt and G. Dall'Agata, *Vacua of $N = 2$ gauged supergravity derived from non-homogenous quaternionic spaces*, *Nucl. Phys.* **B 627** (2002) 357 [hep-th/0112136].

[20] C. Csáki, J. Erlich, T.J. Hollowood and Y. Shirman, *Universal aspects of gravity localized on thick branes*, *Nucl. Phys.* **B 581** (2000) 309 [hep-th/0001033].

[21] M. Gremm, *Thick domain walls and singular spaces*, *Phys. Rev.* **D 62** (2000) 044017 [hep-th/0002040].

[22] E.E. Flanagan, S.H.H. Tye and I. Wasserman, *Brane world models with bulk scalar fields*, *Phys. Lett.* **B 522** (2001) 155 [hep-th/0110070].

[23] S. Kobayashi, K. Koyama and J. Soda, *Thick brane worlds and their stability*, *Phys. Rev.* **D 65** (2002) 064014 [hep-th/0107025].

[24] A. Wang, *Thick de Sitter brane worlds, dynamic black holes and localization of gravity*, *Phys. Rev.* **D 66** (2002) 024024 [hep-th/0201051].

[25] N. Sasakura, *A de Sitter thick domain wall solution by elliptic functions*, *J. High Energy Phys.* **02** (2002) 026 [hep-th/0201130].

[26] N. Sasakura, *Analytic continuations of de Sitter thick domain wall solutions*, *Phys. Rev.* **D 66** (2002) 065006 [hep-th/0203032].

[27] O. Castillo-Felisola, A. Melfo, N. Pantoja and A. Ramirez, *Localizing gravity on exotic thick 3-branes*, *Phys. Rev.* **D 70** (2004) 104029 [hep-th/0404083].

[28] I. Oda, *Locally localized gravity models in higher dimensions*, *Phys. Rev.* **D 64** (2001) 026002 [hep-th/0102147].

[29] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[30] Supernova Cosmology Project collaboration, S. Perlmutter et al., *Measurements of omega and lambda from 42 high-redshift supernovae*, *Astrophys. J.* **517** (1999) 565 [astro-ph/9812133].

[31] Supernova Search Team collaboration, A.G. Riess et al., *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, *Astron. J.* **116** (1998) 1009 [astro-ph/9805201].

[32] F.A. Brito, M. Cvetič and S. Yoon, *From a thick to a thin supergravity domain wall*, *Phys. Rev.* **D 64** (2001) 064021 [hep-ph/0105010].

[33] M. Cvetič and H.H. Soleng, *Supergravity domain walls*, *Phys. Rept.* **282** (1997) 159 [hep-th/9604090].

[34] M. Cvetič, *Domain wall world(s)*, *Int. J. Mod. Phys.* **A 16** (2001) 891 [hep-th/0012105].

[35] A. Miemiec, *A power law for the lowest eigenvalue in localized massive gravity*, *Fortschr. Phys.* **49** (2001) 747 [hep-th/0011160].

[36] M.D. Schwartz, *The emergence of localized gravity*, *Phys. Lett.* **B 502** (2001) 223 [hep-th/0011177].

[37] R. Gregory, V.A. Rubakov and S.M. Sibiryakov, *Opening up extra dimensions at ultra-large scales*, *Phys. Rev. Lett.* **84** (2000) 5928 [hep-th/0002072].

[38] G.R. Dvali, G. Gabadadze and M. Porrati, *Metastable gravitons and infinite volume extra dimensions*, *Phys. Lett.* **B 484** (2000) 112 [hep-th/0002190].

[39] G.R. Dvali, G. Gabadadze and M. Porrati, *A comment on brane bending and ghosts in theories with infinite extra dimensions*, *Phys. Lett.* **B 484** (2000) 129 [hep-th/0003054].

[40] D. Zwillinger, *Handbook of differential equations*, Academic Press, New York 1998.

[41] D. Meade, *Differential equations*, http://www.mapleapps.com.

[42] A. Hegarty, *Numerical solutions of nonlinear algebraic equations*, http://www.ul.ie/~ahegarty/MS4014/notes.pdf.

[43] M. Mueller, *Solution of the 1-D Schroedinger equation*, http://www.mapleapps.com.

[44] I. Brevik, K.A. Milton, S. Nojiri and S.D. Odintsov, *Quantum (in)stability of a brane-world AdS_5 universe at nonzero temperature*, *Nucl. Phys.* **B 599** (2001) 305 [hep-th/0010205].

[45] L. Dolan and R. Jackiw, *Symmetry behavior at finite temperature*, *Phys. Rev.* **D 9** (1974) 3320.

[46] S. Weinberg, *Gauge and global symmetries at high temperature*, *Phys. Rev.* **D 9** (1974) 3357.

[47] A. Vilenkin, *Gravitational field of vacuum domain walls and strings*, *Phys. Rev.* **D 23** (1981) 852.

[48] W.D. Goldberger and M.B. Wise, *Modulus stabilization with bulk fields*, *Phys. Rev. Lett.* **83** (1999) 4922 [hep-ph/9907447].

[49] S. Thambyahpillai, *A closer look at two AdS_4 branes in an AdS_5 bulk*, hep-th/0409190.

[50] D. Bazeia, C. Furtado and A.R. Gomes, *Brane structure from scalar field in warped spacetime*, *JCAP* **02** (2004) 002 [hep-th/0308034].

[51] D. Bazeia and A.R. Gomes, *Bloch brane*, *J. High Energy Phys.* **05** (2004) 012 [hep-th/0403141].

[52] A. Campos, *Critical phenomena of thick branes in warped spacetimes*, *Phys. Rev. Lett.* **88** (2002) 141602 [hep-th/0111207].

[53] D. Bazeia, F.A. Brito and J.R.S. Nascimento, *Supergravity brane worlds and tachyon potentials*, *Phys. Rev.* **D 68** (2003) 085007 [hep-th/0306284].